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Jacobsen, H. Moust

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H. Moust Jacobsen, University of Aalborg

Basic Phenomena in Cyclic Behaviour of Soil

Abstract

A simple set of formulas can describe the most common phenomena in the behaviour of soil subjected to alternating loads. The behaviour can be characterized as "stabilization", "cyclic mobility" or "liquefaction". The equations for the hysteretic stress-strain curve during cyclic loading and reloading make it possible to determine the accumulated mechanical energy dissipated in the soil. In a simplified situation the accumulated energy can be used as a damage indicator.

Introduction

The earthquakes in 1964 in Niigata, Japan and Alaska, USA resulted in great damages on buildings and harbour constructions. For the first time it was recognized that alternating movements in fine sand can rise the pore water pressure, which reduces the effective in situ stresses and the strength of the soil, but increases the movements in the soil. The soil may even loose its shear strength totally and it will then liquefy.

In Niigata pore pressures grew up in a certain depth, but upwards gradients caused liquefication near the soil surface 10 minutes later.

When the oil fields were discovered in the North Sea and soil investigations for the large off shore structures were carried out a few years later, the soil showed to a large extent to consist of fine sands with grain size distribution curves almost similar to those in Niigata. The North Sea is not a region for earthquakes, but very strong storms can be expected several times every year with waves of heights of 20 - 30 meters.

Since then a very intensive study of the pore pressure build up has taken place all around the world and a great deal of quantitative theories for the prediction of soil behaviour during alternating loads have been presented. Two different basic assumptions are:

i) Alternating loads build up pore pressure and liquefaction will develop if the amplitude or the number of cycles are big enough. (Initiated by Seed and Lee in 1966).

ii) The initial effective stress state of the soil plays a definitive role in the behaviour of the soil. If the initial shear stress exceeds a certain value an alternating load will reduce the pore pressure and a stable situation will be created, e.g. Castro and Poulus (1977), Loung (1980).

The purpose of this paper is to show a mathematical formulation of such phenomena as pore pressure build up, liquefaction, cyclic mobility, hysteresis and dissipation of energy. It is based on test series of cyclic triaxial tests on sand executed at the University of Aalborg. Some early results have already been described, e.g. Jacobsen and Ibsen (1989).

The test material is a uniform sand called Lund No 0. The mean diameter $d_{50} = 0.4$ mm, the coefficient of uniformity $U = 1.7$, the void ratio $e = 0.63$ corresponding to a density index $I_D = 0.7$. The test specimens were prepared by the pluviale method and carefully saturated at a total vacuum.

Cyclic tests

The quantities measured during cyclic tests are the total mean normal stress p, the deviation stress $q = \sigma_1 - \sigma_3$, the pore pressure u and the vertical relative deformation ϵ_1.

After consolidation at an initial stress state $(q_0,\ p_0')$ the drains are closed and the specimen subjected to shear stresses with constant amplitude a, as shown in Fig. 1a. During a simple loading cyclus the mean normal stress p' varies only a little corresponding to a small but systematic increment in pore pressure. During a number of cycles p' varies as indicated in Fig. 1a with the thin-lined box. If the failure condition is satisfied only once (point L) the effective mean pressure will vary much more than earlier observed and may even drop to zero. This behaviour is called liquefaction. The strain amplitude grows very quickly and when 20% is reached the complete liquefaction occurs (Castro). In fact the test equipment looses control completely. In the corresponding situation all structures on the surface of a sand layer will be damaged during an earthquake.

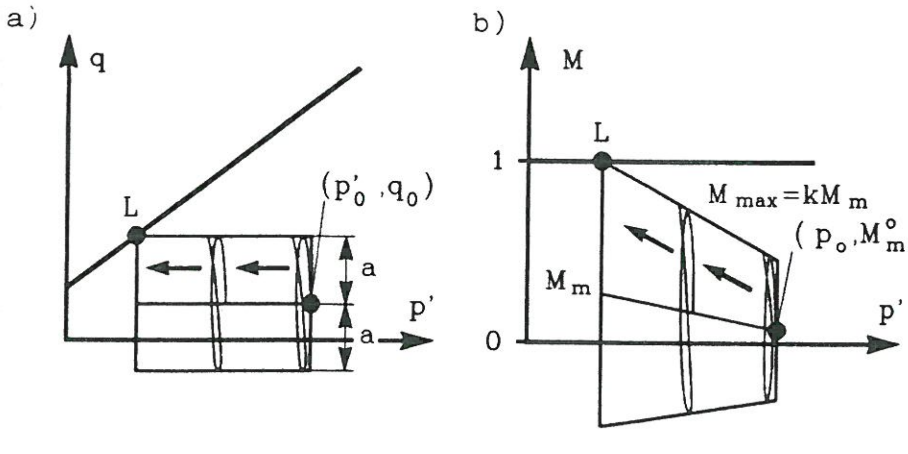

Figure 1: Cyclic loading and liquefaction. b) normalized with respect to q_f.

Mobilization Index M

The deviation stress q can be normalized by introducing a mobilization index M by

$$M = q/|q_f| \qquad ; \qquad -1 < M < 1 \tag{1}$$

where q_f is the deviation stress q corresponding to the drained failure state at the actual mean normal stress p'. The advantage of using a mobilization index instead of a stress ratio $(\sigma_1 - \sigma_3)/\sigma_3$ is obvious: It is possible to compare tests with different soil and densities because q is normalized with respect to the strength of the soil.

The mobilization index is introduced in Fig. 1b. The drained stress state $(p'_0,\ q_0)$ before cyclic loading is then $(p'_0,\ M^{\circ}_m)$. M_{max} is the maximum value of M at each cycle.

If the amplitude a is constant it is seen that

$$M_{max} = \frac{q_0 + a}{q_0} M_m = k M_m \tag{2}$$

where k is the amplitude ratio and M_m is the mean value of M.

If $|M_{max}| = 1$ only once, liquefaction will occur.

Triaxial tests

The Coulomb failure criterion is not symmetric in triaxial tests.

$$\begin{aligned} &\text{In compression:} \quad q_f = \frac{6 sin\phi'}{3 - sin\phi'}\, p' \\ &\text{In extension:} \quad q_f = \frac{-6 sin\phi'}{3 + sin\phi'}\, p' \end{aligned} \tag{3}$$

for a cohesionless soil ($c' = 0$). If q varies symmetrically, large deformations will develop during extension and failure by "necking" may take place. Necking reduces the strength parameters.

From (3) follows that if $M^f_m = sin\phi/(3 + sin\phi)$ at failure, then failure takes place simultaneously in compression and extension (see Fig. 2). The initial value of M^{of}_m corresponding to this situation is given by

$$M^{of}_m = \frac{1}{18}(3 - sin\phi)\frac{a}{p'_0} \tag{4}$$

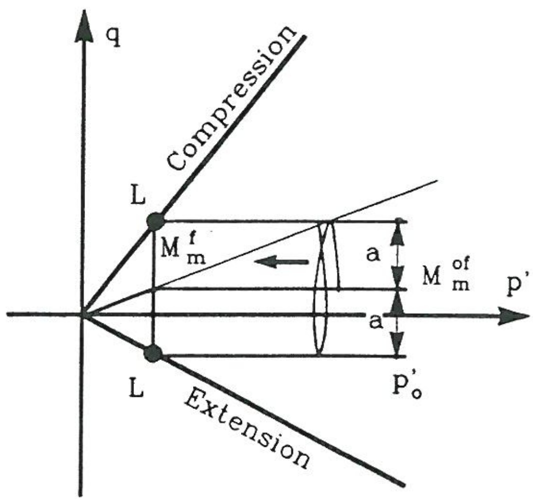

Figure 2: Liquefaction in compression and extension.

In cyclic triaxial tests this value of M_m° is a lower limit for M_m° in order to prevent necking.

In tests with $M_m^\circ = M_m^{of}$ the strains in loading and reloading are almost identical and the cyclic stress-strain curves are reversible.

The stable state M_s

It is now postulated that a stable state exists at a certain mobilization index M_s. If $M_s = M_m$ and $| M_{max} | < 1$ then the positive pore pressure generated during loading equals the negative pore pressure generated during unloading (Risborg Madsen, 1987).

If $M_m > M_s$ and $| M_{max} | < 1$ then negative pore pressure will develop and the effective stress level will increase until the stable state is reached. This phenomenon is called "stabilization" (sequence C in Fig. 3). If $M_m < M_s$ and $| M_{max} | < 1$ a positive pore pressure will be built up and the effective stress level will fall until the stable state is reached. This phenomenon is called "cyclic mobility" (sequence A in Fig. 3).

It is seen that $M_m < M_s$ may lead to liquefaction after a number of cycles if the amplitude ratio k exceeds a critical value k_c (sequence B in Fig. 3). But if $M_m > M_s$ failure must occur during the first cyclus or a stabilization will take place.

The stable state M_s in cyclic loading must not be confounded with the "characteristic state CL" as defined by Loung (1980). In triaxial tests with "static" loading and constant cell pressure the volumetric strain changes from compression to dilatation when q passes the characteristic state. The corresponding value of M is 0.8 - 0.9.

As shown in Fig. 3 a cyclic loading sequence will end up with a mean mobilization index $M_m = M_s$ if the number of cycles N is large enough and if M_{max} does not exceed 1. If $M_m = M_s$ a stable situation exits even if the number of cycles became very large.

A simple formulation of this theory is given by:

$$\begin{aligned} M_m &= M_m^\circ + (M_s - M_m^\circ)\, f(N) \\ M_{max} &= k \cdot M_m \leq 1 \end{aligned} \tag{5}$$

where k is the amplitude ratio defined in formula (2).

$f(N)$ is a function of the number of cycles. $f(N) = 0$ for $N = 0$ and $f(N) \rightarrow 1$ for $N \rightarrow \infty$. A possible expression may be

$$f(N) = \left(\frac{N}{N + N_0}\right)^l \tag{6}$$

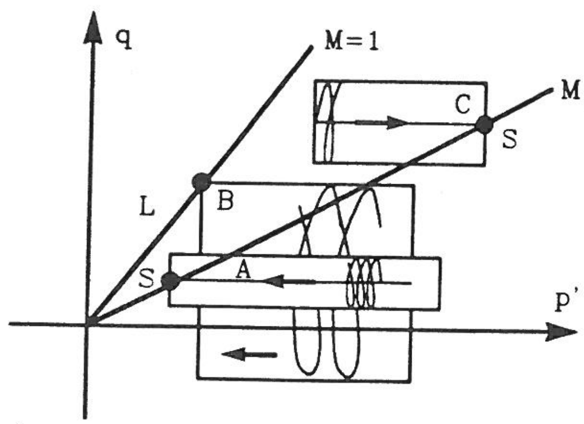

Figure 3: Loading sequences causing a) cyclic mobility. b) liquefaction. c) stabilization.

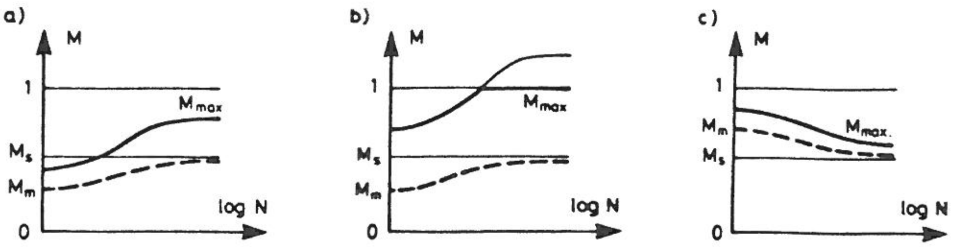

Figure 4: Development of the degree of mobilization under cyclic loading.
a: Increasing M_{max} resulting in the stable state.
b: Increasing M_{max} resulting in liquefaction.
c: Increasing M_{max} resulting in the stable state.

where N_0 depends on M_m°. l is rather close to 1. In analysis of liquefaction risks during earthquakes an advantageous value of l is 1.25. From triaxial tests is estimated

$$N_0 M_m^\circ \approx 5 \tag{7}$$

Test results have already been published (Jacobsen and Ibsen, 1989).

Fig. 4 shows the three possible developments of M_m and M_{max} during cyclic loading as determined by formula (5).

Hysteretic behaviour of soil

Two stress cycles from the same sequence are shown in Fig. 5a. It shows that the behaviour of the sand is strongly hysteretic, but only small irreversibility occurs.

The irreversibility causes permanent deformations. It depends very much on M_m° and does not occur in tests with $M_m^\circ = M_m^{of}$, but with $M_m^\circ \gg M_m^{of}$ the permanent deformations play a major role. In earthquakes no permanent horizontal movements are developed in a systematic way and therefore no irreversibilities occur. $M_m^\circ \approx M_m^{of}$ and earthquakes may cause liquefaction in some parts of the soil layer. However, if a foundation is subjected to wave loads in heavy storms, the average value of M_m beneath the foundation normally equals or even exceeds M_s. Liquefaction is normally not the problem but large permanent settlements may take place.

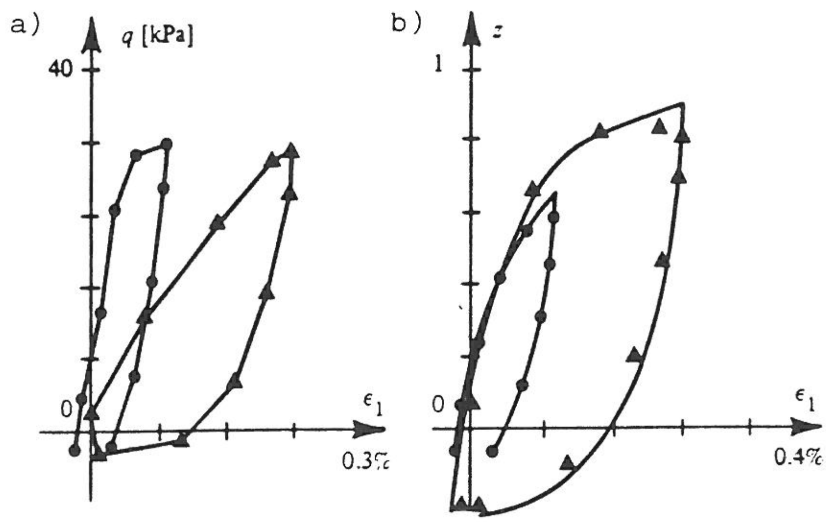

Figure 5: Hysteretic curves estimated from eq. (10), and measured in triaxial tests.

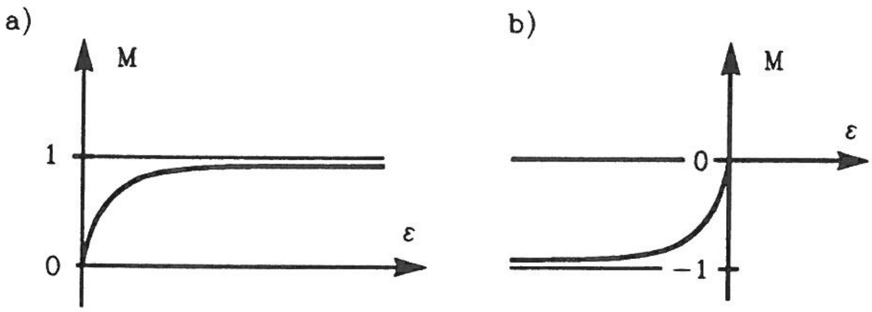

Figure 6: Normalized performance curves in undrained state.

In this paper only the hysteretic behaviour of sand is mentioned in details. The curves at Fig. 5a seem to be very complicated for a mathematical description. However, by introducing the mobilization index M (Fig. 5b) the curves became very regular. During a test the pore pressure built up causing the decrease of the shear modulus G. The mobilization index M increases (formula 5), but the normalized shear module G_M is nearly a constant.

A formula for the hysteretic curve can be argued as follows:

The first loading in an undrained test can be described by

$$\frac{\partial M}{\partial \epsilon} = G_M\,(1 - M^n) \tag{8}$$

where n is a number which describes the curvature (Fig. 6a). In unloading nearly the same formula can be used

$$\frac{\partial M}{\partial \epsilon} = G_M\,(1 - |M|^n)$$

A hysteretic cyclic curve could then be described by:

$$\frac{\partial M}{\partial \epsilon} = G_M\,(1 - sign\,(\frac{M}{d\epsilon})\,|M|^n) \tag{9}$$

It shows continuity and differentiability for $M = 0$ (Fig. 7).

A further study of formula (9) shows that an unrealistic irreversibility occurs when $M_m \neq 0$ except for small stress amplitudes. In order to separate the hysteretic behaviour from irreversibility, formula (9) is modified:

$$\frac{\partial\,(M - M_m)}{\partial \epsilon} = G_M\,(1 - sign\,(\frac{M - M_m}{d\epsilon})|\frac{M - M_m}{1 - M_m}|^n) \tag{10}$$

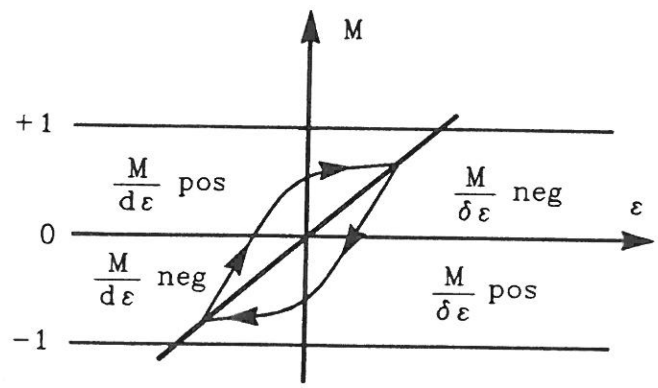

Figure 7: Hysteretic curve.

This formula is assumed to describe the hysteretic behaviour in triaxial tests as close as possible. Characteristic values of G_M and n are found from triaxial tests:

$$G_M = 900 \quad \text{and} \quad n = 0.5$$

Liquefaction during earthquakes

During an earthquake shear waves (SH-waves) propagate from the stiff subsoil upwards through the weaker layer near the soil surface.

Shear stresses act on horizontal planes with irregular varying magnitude and direction. In the horizontal and vertical planes no elongation takes place. The volume change is zero due to undrained circumstances and the absense of compression waves. The Mohr circles for strain and stresses are shown in Fig. 8. It is seen that $\sigma_x' = \sigma_z'$, $\gamma_{xz} = \epsilon_1$ and $q = 2\ \tau_{xz}$.

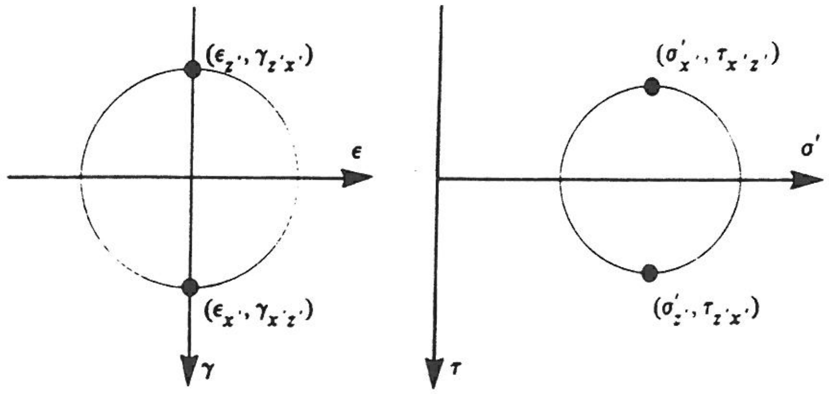

Figure 8: Mohrs circles for strains and stresses in a soil element during an earthquake.

The dissipated energy per unit volume of soil is then given by:

$$\dot{E} = \sigma_x' \cdot \dot{\epsilon_x} + \dot{\epsilon_x} + \sigma_z' \cdot \dot{\epsilon_z} + 2\ \tau_{xz} \cdot \dot{\gamma_{xz}} = q \cdot \dot{\epsilon_1} \tag{11}$$

If we assume that the difference between field conditions (plane state, rotating principal stresses) and triaxial tests (axial state, fixed principal stresses) are unimportant, triaxial tests can be used to determine the dissipated mechanical energy. Since irreversible movements do not take place during an earthquake, only energy due to the hysteretic behaviour of the soil should be taken into account.

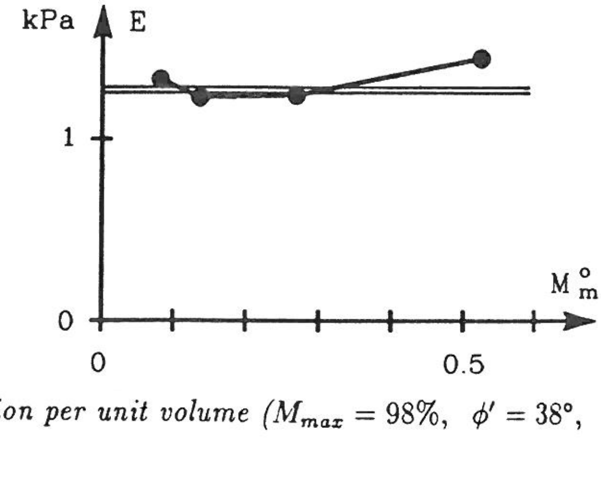

Figure 9: Energy dissipation per unit volume ($M_{max} = 98\%$, $\phi' = 38°$, $c' = 0\ kPa$, $p' = 25\ kPa$).

The dissipated accumulated energy E under cyclic loading can be estimated from equations (5), (6), (7), (10) and (11). At failure it turns out to be nearly constant when using $l = 1.25$ in equation (6) (Fig. 9). When $M_{max} = 0.98$ the energy E_f per unit volume can be assumed to be:

$$E_f = 0.2 \frac{sin\phi'}{3 - sin\phi'} p'$$

If E_f is supposed to be constant even when the amplitude varies, E_f can be used as a damage index. It means that if the irregular loading history of a soil element during an earthquake is estimated then the risk of liquefaction can be determined in a very simple way by calculating the absorbed mechanical energy and compare it with E_f. This method is very uncomplicated but it shows the same tendency as more sophisticated methods: The influence from variations at high deviation stresses is very important compared with the influence from many small loading cycles.

Conclusion

Based on cyclic triaxial tests the behaviour of a sand subjected to alternating load is studied. The terms "cyclic mobility", "stabilization" and "liquefaction" are defined and formulas concerning the development in pore pressure and the safety against failure are proposed. The hysteretic behaviour of sand is also described and it is proposed to use the accumulated mechanical energy per unit volume as a damage indicator when analyzing the risk of liquefaction during earthquakes.

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